Full Implementation of an Implicit Nonlinear Model with Memory in an Harmonic Balance Software

R. Sommet and E. Ngoya

***Abstract*—This letter describes the mathematical formalism for a full computation of the Jacobian matrix as part of the harmonic balance (HB) technique in the case of implicit nonlinear equations with memory. This new formalism, based on a double Fourier transform of the nonlinearity derivatives versus the time-domain commands, has been developed especially in order to permit the direct coupling of the semiconductor equations of a GaInP/GaAs heterostructure bipolar transistor in a circuit simulator based on the HB analysis technique.**

I. Introduction

CIRCUIT simulators require numerical algorithms to determine the steady-state regimes of microwave circuits. This job essentially consists of computing the solution of a nonlinear system of equations described either with a time-domain method or an harmonic balance (HB) method. Numerous techniques, which can be put into two classes, are presently available. The first one consists of transforming the problem into an optimization problem, whereas the second one is based on directly solving nonlinear equations using a fixed-point iterative method. Most of computer-aided design (CAD) software for microwave nonlinear circuit analysis presently reduce the nonlinear semiconductor devices to equivalent circuit elements with closed-form analytical approximations for the nonlinear elements. These analytical approximation functions are very often explicit, which signifies that they can be written as (1) where y represents the nonlinear function of the command variable x

$$\boldsymbol{y}(t) = \boldsymbol{f_{NL}}\left[\mathrm{x}(t),\, \mathrm{x}(t-\tau),\, \frac{d\mathrm{x}}{dt}, \cdots\right]. \tag{1}$$

The ability to codesign circuits and devices will be a major feature of CAD tools for the design of monolithic microwave integrated circuits (MMIC's) [1]. In this aim, the coupling of circuit simulators and physical simulators appears to be very promising as it consists of replacing the analytical equivalent circuit by device simulators. This coupling may be based either on time-domain or HB methods [2]. Some attempts using the semiconductor equations have been made in the time domain [3] for heterojunction bipolar transistors (HBT's) and, in the case of an HB method, for field-effect transistor (FET) devices [4]. In the case of semiconductor equations, nonlinear functions like current–voltage or capacitance–voltage characteristics are no longer explicitly known. Only an implicit expression of the nonlinearities as (2) is available:

$$\boldsymbol{f_{NL}}\left[\mathrm{x}(t),\, \mathrm{x}(t-\tau),\, \frac{d\mathrm{x}}{dt},\, \mathrm{y}(t),\, \frac{d\mathrm{y}}{dt}\, \cdots\right] = 0. \tag{2}$$

This letter is principally dedicated to the nonlinearity of type (2). The next section describes the mathematical formalism for a full computation of the Jacobian matrix in the case of a Newton–Raphson iterative method in the HB formalism. This technique, which allows to obtain accuracy, computational speed, and excellent convergence properties, has been successfully tested on the coupling of the semiconductor equations of a GaInP HBT to a circuit simulator based on the HB analysis technique.

II. The Jacobian Matrix Computation

The HB analysis technique has been already described in several papers [5]. This numerical method is used to determine the steady-state regime of microwave circuits and has proved to be efficient and accurate. In the frequency domain, the modified nodal analysis equation describing the circuit takes the form

$$\begin{aligned} \boldsymbol{H}(\mathbf{X}) &= \mathbf{X} - \mathbf{A_y}\mathbf{Y}(\mathbf{X}) - \mathbf{A_g}\mathbf{G} \\ &= \mathbf{0} \end{aligned} \tag{3}$$

where $\mathbf{X}$ represents the Fourier transform vector of the commands, $\mathbf{Y}(\mathbf{X})$ the Fourier transform vector of the nonlinearities, $\mathbf{G}$ the Fourier transform vector of the generators, and $\mathbf{A_y}$ and $\mathbf{A_g}$ some linear matrix of elements. Several numerical techniques can be employed to achieve the solution of such a problem [6], [7]. The most employed is the Newton iterative method. In computer implementation, the general Newton iteration is formulated as

$$\mathbf{J_H}(\mathbf{X}^k)(\mathbf{X}^{k+1} - \mathbf{X}^k) = -\mathbf{H}(\mathbf{X}^k). \tag{4}$$

$\mathbf{J_H}$ is the Jacobian matrix of the nonlinear system of equations. If we consider (3), $\mathbf{J_H}$ can be computed as

$$\mathbf{J_H} = \mathbf{1} - \mathbf{A_y}\left[\frac{\partial \mathbf{Y}(\mathbf{X})}{\partial \mathbf{X}}\right] \tag{5}$$

where $\mathbf{1}$ is the identity matrix and $[\partial \mathbf{Y}(\mathbf{X})/\partial \mathbf{X}]$ is the matrix of the nonlinearity derivatives in the frequency domain.

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The general term $\partial \mathbf{Y}_k / \partial \mathbf{X}_l$ of this matrix is given by the derivative of the kth component of the nonlinearity versus the lth component of the command variable. In the case of an explicit form of the nonlinearities, $\partial \mathbf{Y}_k / \partial \mathbf{X}_l$ is given by

$$\frac{\partial Y_k}{\partial X_l} = \frac{1}{N} \sum_{p=0}^{N-1} \left[\frac{\partial y(t)}{\partial x(t)} + e^{-jl\omega_0 \tau} \frac{\partial y(t)}{\partial x(t-\tau)} + \cdots \right] \cdot e^{-j(k-l)\omega_0 p \Delta t}. \tag{6}$$

We recognize in (6) the $(k-l)$th term of the discrete Fourier transform (DFT) of

$$\left[\frac{\partial y(t)}{\partial x(t)} + e^{-jl\omega_0 \tau} \frac{\partial y(t)}{\partial x(t-\tau)} + \cdots \right].$$

Our interest here is principally in the computation of the solution with nonlinearities expressed by (2). Indeed, If we consider an HBT physical simulator, the nonlinear variables are the carrier densities n and p, the potential φ, and the current densities Jn and Jp. In the case of drift diffusion (DD) equations, two continuity equations link dn/dt, dp/dt, and the current densities Jn, Jp. The presence of the time-domain derivatives of n and p in these equations expresses a memory effect. The boundary conditions on the electrostatic potential φ are expressed by the command vector $x(t)$. Also, it provides us a current–voltage relationship in the implicit form as (2). Time-domain discretization of the DD equations and the solving of the nonlinear system gives the values of the nonlinearity vector y as a function of the command vector x where

$$y = \{y(t_0) \cdots y(t_{N-1})\} \tag{7a}$$

$$x = \{x(t_0) \cdots x(t_{N-1})\}. \tag{7b}$$

Simultaneously, the derivatives $\partial y(t_n)/\partial x(t_m)$ can be obtained either analytically or either by numerical differentiation. In the case of an implicit form

$$\frac{\partial Y_k}{\partial X_l} = \frac{\partial}{\partial X_l} \left[\frac{1}{N} \sum_{p=0}^{N-1} y(p) e^{jk\omega_0 p \Delta t} \right] \tag{8}$$

$$= \frac{1}{N} \sum_{p=0}^{N-1} \frac{\partial y(p)}{\partial X_l} e^{jk\omega_0 p \Delta t} \tag{9}$$

or $y(p)$ is a function of all the time-domain command samples, so

$$\frac{\partial y(p)}{\partial X_l} = \sum_{m=0}^{N-1} \frac{\partial y(p)}{\partial x(p-m)} \frac{\partial x(p-m)}{\partial X_l} \tag{10}$$

$$\frac{\partial Y_k}{\partial X_l} = \frac{1}{N} \sum_{p=0}^{N-1} \left[\sum_{m=0}^{N-1} \frac{\partial y(p)}{\partial x(p-m)} \frac{\partial x(p-m)}{\partial X_l} \right] \cdot e^{-jk\omega_0 p \Delta t} \tag{11}$$

or

$$x(k) = \sum_{n=0}^{N-1} X_n e^{jk\omega_0 k \Delta t} \tag{12}$$

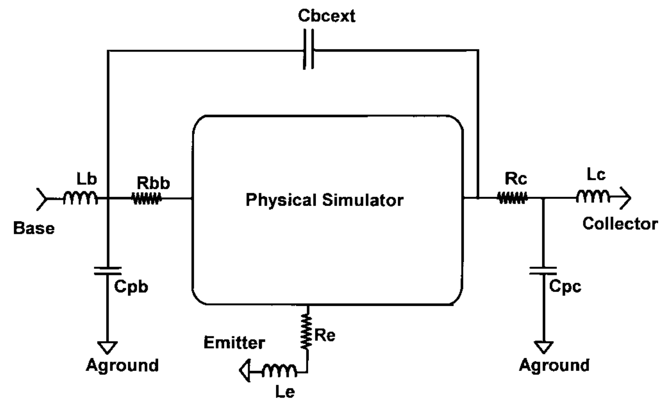

Fig. 1. Schematic of the HBT model.

so

$$\frac{\partial x(p-m)}{\partial X_l} = e^{jl(p-m)\Delta t}. \tag{13}$$

Writing (13) in (11), we obtain

$$\frac{\partial Y_k}{\partial X_l} = \frac{1}{N} \sum_{p=0}^{N-1} \left[\sum_{m=0}^{N-1} \frac{\partial y(p)}{\partial x(p-m)} e^{jl(p-m)\omega_0 \Delta t} \right] \cdot e^{-jk\omega_0 p \Delta t}. \tag{14}$$

We define also $u_m(p) = \partial y(p)/\partial x(p-m)$ as a function of p with m constant. Then, (14) becomes

$$\frac{\partial Y_k}{\partial X_l} = \frac{1}{N} \sum_{m=0}^{N-1} N \left[\frac{1}{N} \sum_{p=0}^{N-1} u_m(p) e^{-j\omega_0 (k-l) p \Delta t} \right] \cdot e^{-jlm\omega_0 \Delta t} \tag{15}$$

or $(1/N) \sum_{p=0}^{N-1} u_m(p) e^{-j\omega_0 (k-l) p \Delta t}$ represents the $(k-l)$ term of the DFT of $u_m(p)$ written $U_m(k-l)$.

Finally, $\partial Y_k / \partial X_l = (1/N) \sum_{m=0}^{N-1} N U_m(k-l) e^{-jl\omega_0 m \Delta t}$. This expression exhibits the lth term of the DFT of $NU_m(k-l)$ written $\hat{U}(k-l, l)$.

The exact Jacobian matrix of the nonlinear system is obtained by application of a double Fourier transform to the nonlinearity derivatives versus time-domain commands.

III. Results

This previous method has been successfully applied to the direct coupling of the semiconductor equations of a GaInP/GaAs HBT in our laboratory HB simulator. The devices, manufactured at the Thomson LCR foundry [8], have been used for the optimization of a class B power amplifier for mobile communication at 1.8 GHz. The HBT device represented in Fig. 1 is biased through a $\lambda/4$ transmission line which allows to short circuit the 2^d harmonic of the collector voltage. Optimization of the amplifier becomes feasible in terms of device characteristics or embedding impedances for a particular device as the calculation of the steady-state regime takes about 20 min per power point using an HP712/60 workstation. In Fig. 2 we have represented the results of the amplifier in terms of embedding impedances. Current waveform as well as the load line in the output plane of the device are shown in Figs. 3 and 4.

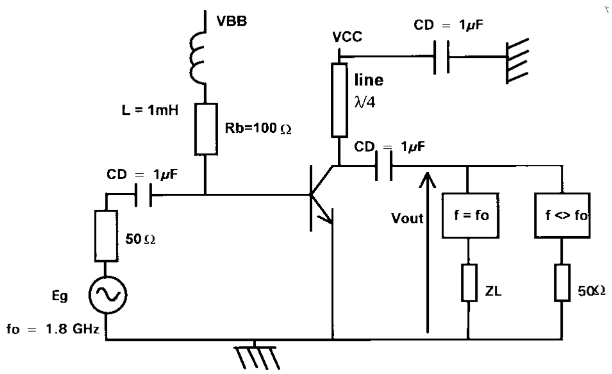

Fig. 2. Schematic of the simulated circuit.

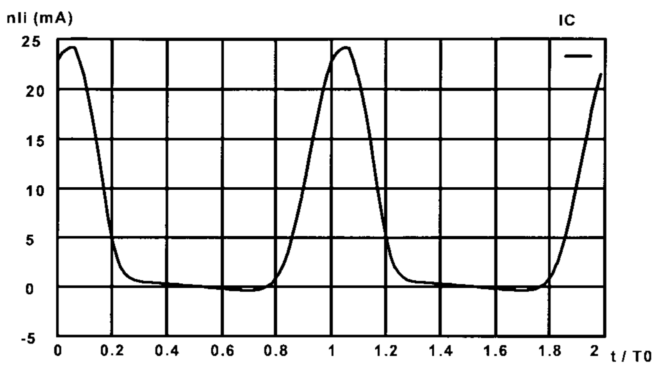

Fig. 3. Temporal collector current waveform IC(t).

IV. Conclusion

A new technique, applied in particular for HB-based circuit analysis, has been presented. More than being another technique which provides only high computational efficiency and a high rate of convergence, the main advantage of this developed technique is its ability to a treat circuit especially in the case of implicit nonlinearities with memory like currents resulting from a device simulator. It has proved to be very efficient for the calculation of power amplifiers for mobile communication and can be used more generally for getting the solution of a nonlinear system of equations.

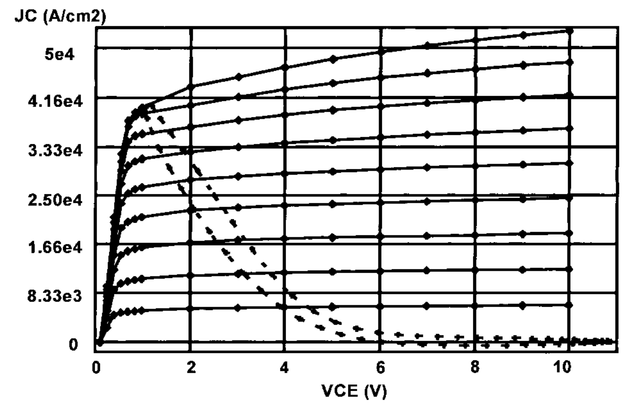

Fig. 4. Load line representing the collector current IC versus the collector-emitter tension VCE.

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